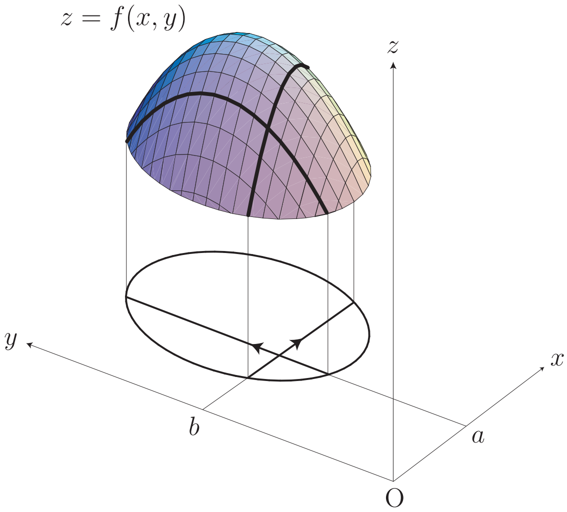
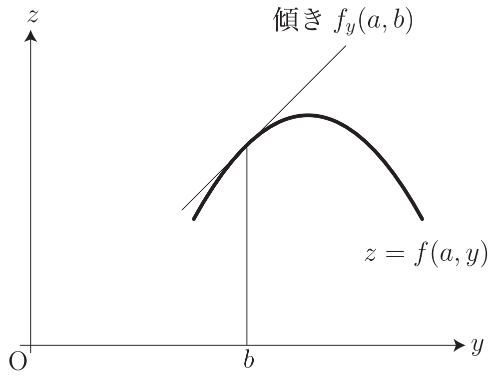
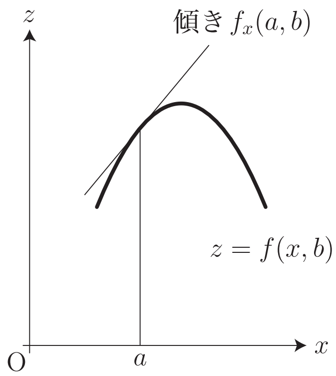
《資料1》２変数関数のグラフ





$x = a$ での切り口



$y = b$ での切り口

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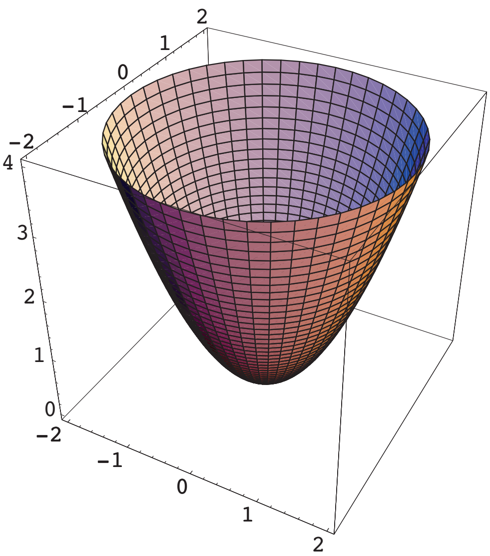
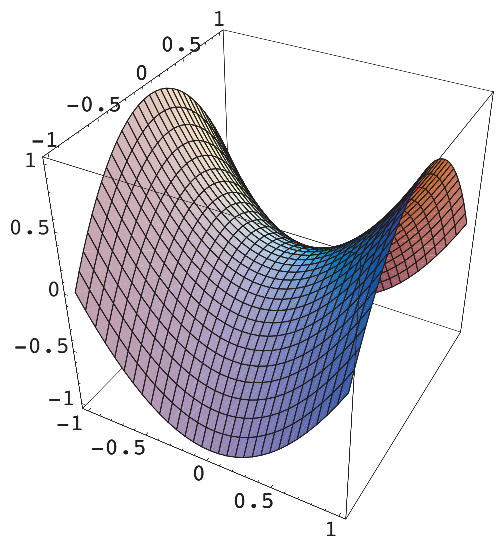
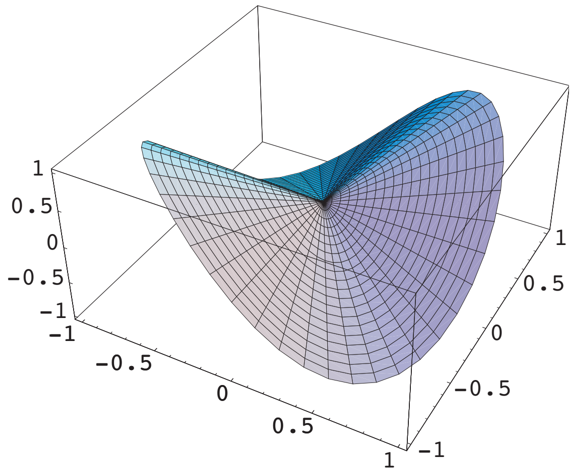
(1) $z = x^2 + y^2$



(2) $z = x^2 - y^2$

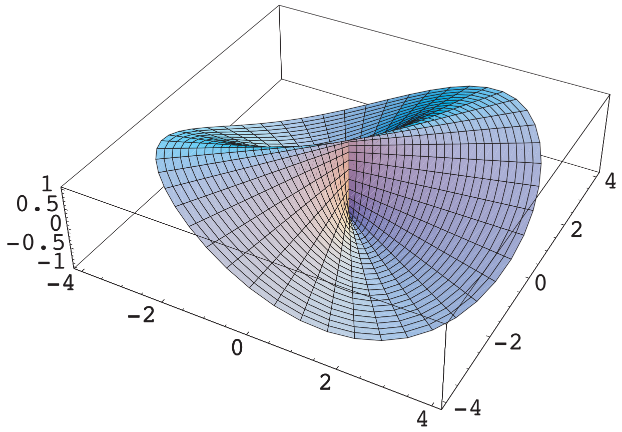


(3) $z = \begin{cases} \dfrac{2xy}{\sqrt{x^2+y^2}} & ((x,y) \neq (0,0)) \\ 0 & ((x,y) = (0,0)) \end{cases}$



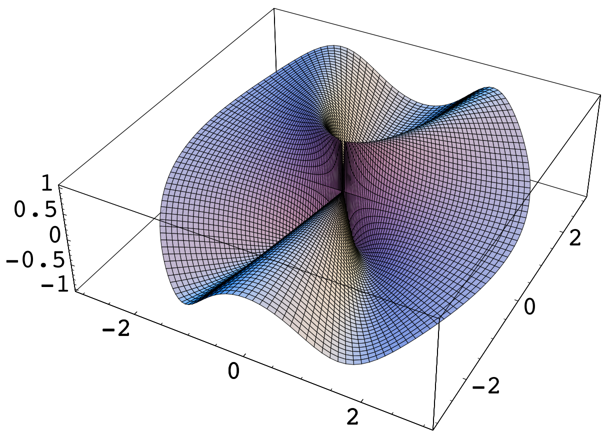
原点において連続かつ偏微分可能だが, 全微分可能でない. (x 軸, y 軸以外の方向には微分可能でない.)

(4) $z = \begin{cases} \dfrac{2xy}{x^2+y^2} & ((x,y) \neq (0,0)) \\ 0 & ((x,y) = (0,0)) \end{cases}$



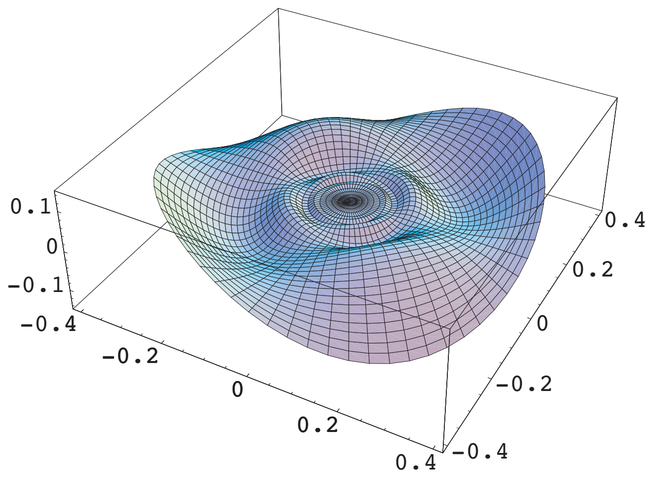
原点において偏微分可能だが連続でない. (x 軸, y 軸以外の原点を通る任意の直線に沿っては原点で連続でない.)

(5) $z = \begin{cases} \dfrac{2x^2y}{x^4+y^2} & ((x,y) \neq (0,0)) \\ 0 & ((x,y) = (0,0)) \end{cases}$



原点を通る任意の直線に沿って, 原点において連続かつ (その直線の方向に) 微分可能. しかし, (2 変数関数としては) 原点において連続ではない.

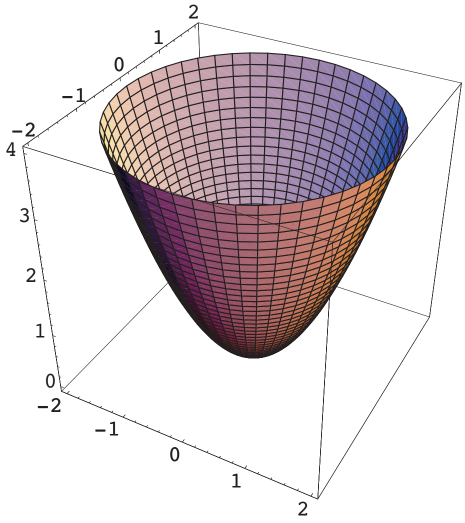
(6) $z = \begin{cases} 2xy\sin\dfrac{1}{\sqrt{x^2+y^2}} & ((x,y) \neq (0,0)) \\ 0 & ((x,y) = (0,0)) \end{cases}$

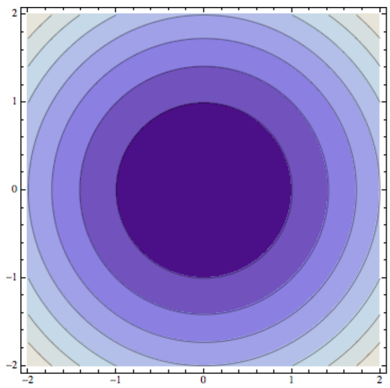


原点において全微分可能であるが, C^1 級ではない.

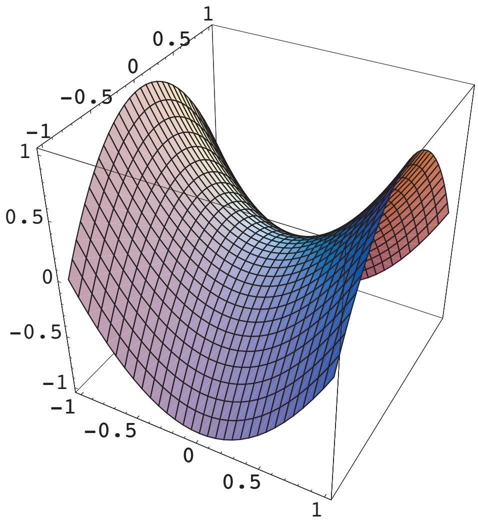
■ グラフと等高線

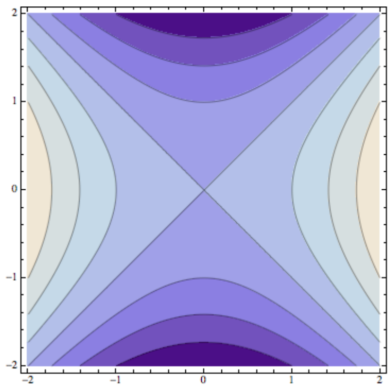
(1) $z = x^2 + y^2$



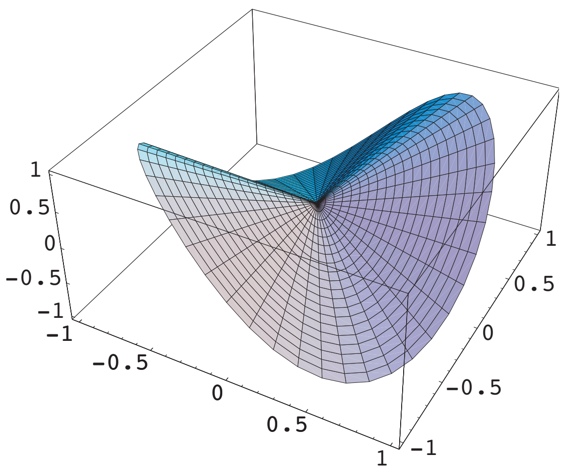


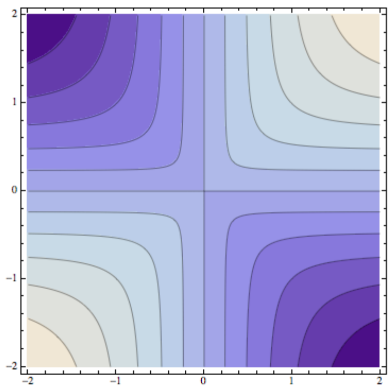
(2) $z = x^2 - y^2$



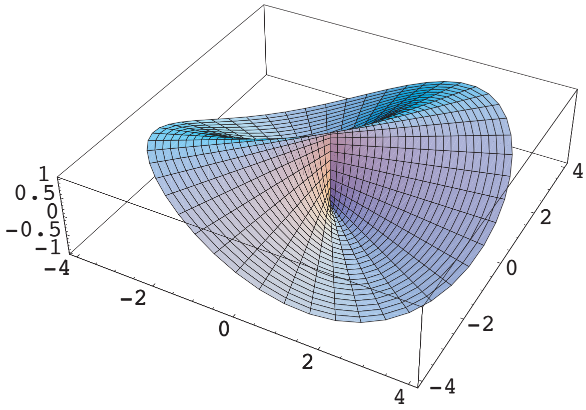


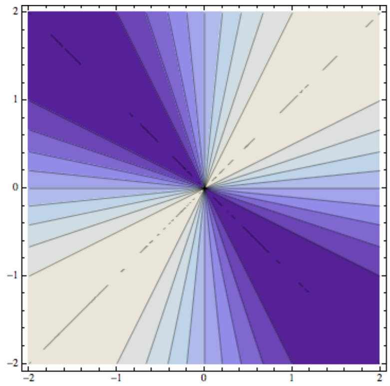
(3) $z = \dfrac{2xy}{\sqrt{x^2 + y^2}}$



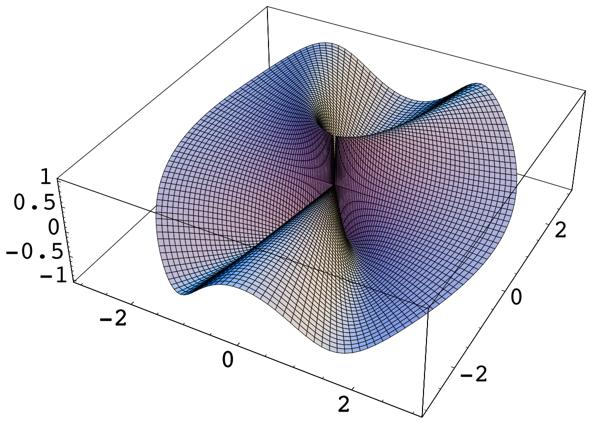


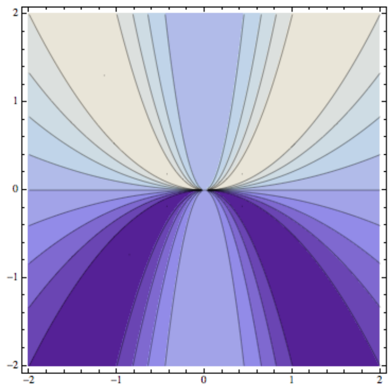
(4) $z = \dfrac{2xy}{x^2 + y^2}$





(5) $z = \dfrac{2x^2 y}{x^4 + y^2}$





(6) $z = 2xy \sin \dfrac{1}{\sqrt{x^2 + y^2}}$